Credit Risk in Banking

CREDIT RISK MODELS

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Merton model

- It consider the financial structure of a company, therefore it belongs to the structural approach models
- Notation:
  - E_t, value of the equity at time t
  - D_t, value of the debt at time t
  - V_t, value of the assets at time t, σ_V its constant volatility
  - T, maturity of the debt

Merton model

By assumption, the value of the asset during the life of the company is equal to the amount of equity plus the debt:

$$V_t = E_t + D_t, \qquad\qquad 0 \leq t < T$$

In T, we declare default if $V_T < D_T$ which means that the asset of the company are not enough to pay the debt.

The assumption of Merton is the following:

- In T,
  - if $V_T \geq D_T$, the shareholders repay the debt
  - if $V_T < D_T$, the shareholders declare bankruptcy and give the whole company as partial repayment of the debt.

It means that the when the shareholders ask for a loan, they also subscribe a put option with strike equal to D_T.

Merton model

Thus, according to the idea that the shareholders buy a put to hedge the credit risk, i.e.

$$D_0 + put = D_T e^{-rT}$$

and then the value of the loan today is

$$D_0 = D_T e^{-rT} - put$$

A further assumption made by Merton is that the value of the asset evolves following a Ito process, i.e.

$$dV_t = \mu_V V dt + \sigma_V V \xi \sqrt{dt}$$

Therefore the evaluation of the put option follows the Black & Scholes formula:

$$D_0 = D_T e^{-rT} - D_T e^{-rT} N(-d_2) + V_0 N(-d_1)$$

Merton model

$$D_0 = D_T e^{-rT} - D_T e^{-rT} N(-d_2) + V_0 N(-d_1)$$
$$D_0 = D_T e^{-rT}\big(1 - N(-d_2)\big) + V_0 N(-d_1)$$
$$D_0 = D_T e^{-rT} N(d_2) + V_0 N(-d_1)$$

Finally we obtain the credit spread:

$$D_T e^{-(r+s)T} = D_T e^{-rT} N(d_2) + V_0 N(-d_1)$$
$$s = -\frac{1}{T} \ln N(d_2) + \frac{V_0}{D_T e^{-rT}} N(-d_1)$$

And we know that the exercise probability is the default probability

$$P(V_T < D_T) = N(-d_2)$$

Merton model

We can compute the default probability for any arbitrary T for which the company has a loan. And thus we observe a probability default term structure.

From empirical observation we have that:

- Companies with a high probability of default has a decreasing term structure
  - i.e. if they survive the first years is more likely they will survive the next
- Companies with a low probability of default has an increasing term structure
  - i.e. even if they are good today, the future is uncertain

Merton model

Pros	Cons
• It shows the main variables: leverage and volatility • Structural approach	• Simplified debt structure and possibility to default only in T • Gaussian distribution assumption • Input variables (V_0 and σ_0) not easy to observe • Risk free rate constant over time • No arbitrage assumption • B&S assumes continuous negotiation of the underlying • No downgrading risk

Longstaff e Schwarts (1995) – Default during the lifetime if V_t is below a threshold
Kim, Ramaswamy e Sundaresan(1993) – Stochastic risk free rate

KMV model

Kealhofer, McQuown and Vasicek – Moody's

- It consider the financial structure of a company, therefore it belongs to the structural approach models
- Notation:
  - E_t, value of the equity at time t, σ_E its constant volatility
  - D_t, value of the debt at time t
  - V_t, value of the assets at time t, σ_V its constant volatility
  - T, maturity of the debt

KMV model

KMV model moves from the Merton model.

- The further observation is that the equity value can be seen as a call option on the assets of a company. Indeed, in T,
  - if $V_T \geq D_T$, the equity value equals the asset minus the debt
  - if $V_T < D_T$, the shareholders declare bankruptcy and the equity value is equal to zero.

$$E_T = \max(V_T - D_T, 0)$$

Then

$$E_0 = V_0 N(d_1) - D_T e^{-rT} N(d_2)$$

Moreover

$$\sigma_E E_0 = \sigma_V V_0 N(d_1)$$

KMV model

$$\begin{cases} E_0 = V_0 N(d_1) - D_T e^{-rT} N(d_2) \\ \sigma_E E_0 = \sigma_V V_0 N(d_1) \end{cases}$$

Solving the system we obtain σ_V and V_0 and we delate one of the drawbacks of Merton model.

KMV partially solve the Merton's simplified debt structure considering both short term debts (b) and long term debt (l) and defining the Default Point

$$DP = b + 0.5l$$

Finally the Distance to Default is defined as

$$DD = \frac{V_0 - DP}{\sigma_V V_0}$$

The probability that the value of the asset will go below the DD and then there will be a default, is simply given by $N(-DD)$

KMV model

An alternative way to compute the probability of default is to consider a database of historical observations.

Then, for each company of the database, we compute the DD and for companies with similar DD we observe how many of them declared bankruptcy.

In this case, the probability of default is called Empirical Default Frequency (*EDF*)

KMV model

Pros	Cons
• EDF and DD can be updated more often than the rating grade • In rating grade approach, companies with same rating share the same probability to default • Debt structure is not oversimplified • Input data are more easy to define	• Gaussian distribution assumption on the equity process • Risk free rate constant over time • No arbitrage assumption • The company must be listed in a market • Market assumed to be efficient

Credit $V@R$ model

We need to briefly recall the concept of Gaussian copula.

We want to find the correlation between two variables V_1, V_2 for which we know the marginal but not the joint distribution.

- We transform V_1 in normal variable U_1 percentile by percentile
- We transform V_2 in normal variable U_2 percentile by percentile
- We assume U_1 and U_2 follow a bivariate normal distribution with correlation coefficient ρ.

Credit $V@R$ model

The two variables for which we want to find the correlation are T_1, T_2 that correspond to the time to default of two companies.

Such variables have cumulative distribution $\mathrm{Q}(T_i)$, i.e. $\mathrm{Q}(T_i) = P(T_i < t)$.

Then the normal distribution U_i is given by

$$P(T_i < t) = P(U_i < u)$$

$$u = N^{-1}(Q(T_i))$$

We repeat the process for both T_1, T_2 and once we have two normal marginal we can find their correlation.

Credit $V@R$ model

Very often the correlation structure is described with a factorial model

$$U_i = a_i F + \sqrt{1 - a_i^2} Z_i$$

where F, Z_i are standard normal distribution pairwise independent. Then

$$P(U_i < u|F) = P\left(Z_i < \frac{u - a_i F}{\sqrt{1 - a_i^2}}\right) = N\left(\frac{u - a_i F}{\sqrt{1 - a_i^2}}\right)$$

But since $P(T_i < t) = P(U_i < u)$ and $u = N^{-1}(Q(T_i))$,

$$P(T_i < t|F) = N\left(\frac{N^{-1}(Q(T_i)) - a_i F}{\sqrt{1 - a_i^2}}\right)$$

Credit $V@R$ model

Assume the distribution Q_i of the time to default T_i are equal for all i.

Assume the copula correlation $a_i a_j$ is the same for every couple i, j then $a_i = \sqrt{\rho}$

And

$$P(T_i < t|F) = N\left(\frac{N^{-1}(Q(T_i)) - \sqrt{\rho}F}{\sqrt{1-\rho}}\right)$$

Since F is a standard normal distribution, $P\left(F < N^{-1}(X)\right) = X$

Then, in a $V@R$ point of view, once we fix the probability X, we find the value F such that the probability of default will be no more than the solution of the following

$$N\left(\frac{N^{-1}(Q(T_i)) - \sqrt{\rho}N^{-1}(X)}{\sqrt{1-\rho}}\right)$$

Credit $V@R$ model

Pros	Cons
• It is not a structural model • It considers $V@R$ perspective • It allows to test different types of copulas • The $V@R$ can be measured at different confidence level	• It is not a structural model • It implies the copula approximation • The confidence reflects the transaction matrix probabilities and we need to approximate

CreditMetrics
JP Morgan

It considers variation of the portfolios due to variation of the rating grade

Input needed:

- Rating system
- Transaction matrix
- Risk free term structure
- Credit spread term structure

CreditMetrics

Let's consider a given transaction matrix, and a bond rated BBB.

Knowing the term structure (risk free and credit spread), we can price the bond according to the different rating grade it will reach at a given maturity. And finally define the distribution of the prices.

Rating	Value	Variation	Probability
AAA	109.37	1.82	0.02
AA	109.19	1.64	0.33
A	108.66	1.11	5.95
BBB	**107.55**	**0**	**86.93**
BB	102.02	-5.53	5.3
B	98.1	-9.45	1.17
CCC	83.64	-23.91	0.12
D	51.13	-56.13	0.18

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The expected value of the bond is 107.09 and the standard deviation is 2.99. The difference 107.55-107.09 is the expected loss. The estimated first percentile is 98.1 and the probability that the bond will fall below 98.1 is 1.47%.

Then, the approximated V@R at 99% is:

$$107.09-98.1=8.99$$

CreditMetrics

Let's consider a second bond rated A and repeat the definition of the distribution of the prices.

Rating	Value	Variation	Probability
AAA	106.59	0.29	0.09
AA	106.49	0.19	2.27
A	106.3	0	91.05
BBB	105.64	-0.66	5.52
BB	103.15	-3.15	0.74
B	101.39	-4.91	0.6
CCC	88.71	-17.59	0.01
D	51.13	-55.17	0.06

CreditMetrics

Assuming zero correlation between the two bonds, the joint migration probability are given by the product of the two marginal distributions.

Bond BBB \ Bond AA		AAA	**AA**	A	BBB	BB	B	CCC	D
		0.09	**2.27**	91.05	5.52	0.74	0.6	0.01	0.06
AAA	0.02	0.00	**0.00**	0.02	0.00	0.00	0.00	0.00	0.00
AA	0.33	0.00	**0.01**	0.03	0.02	0.00	0.00	0.00	0.00
A	5.95	0.01	**0.14**	5.42	0.33	0.04	0.04	0.00	0.00
BBB	**86.93**	**0.08**	**1.97**	**79.15**	**4.80**	**0.64**	**0.52**	**0.01**	**0.05**
BB	5.3	0.00	**0.12**	4.83	0.29	0.04	0.03	0.00	0.00
B	1.17	0.00	**0.03**	1.07	0.06	0.01	0.01	0.00	0.00
CCC	0.12	0.00	**0.00**	0.11	0.01	0.00	0.00	0.00	0.00
D	0.18	0.00	**0.00**	0.16	0.01	0.00	0.00	0.00	0.00

CreditMetrics

According to the quantity of bond AA and BBB bought, according to the joint probability, we define the distribution of the portfolio values and we extract the $V@R$ of the portfolio.

In case of correlated bonds it is needed to estimated such correlation and then adapt the joint transaction matrix. Usually the correlation between issuers' equity is adopted.

CreditMetrics model

Pros	Cons
• It uses market data and forward looking estimates • Adopt a market consistent evaluation • It considers not only defaults but also downgrading • It allows an increasing $V@R$ analysis	• Term structure deterministic • Transaction matrix needs to be estimated • Transaction matrix assumed to be constant in time • Probabilities are rating grade based and not single company based • Assets correlations are estimated through equity correlations

Other models

Portfolio manager (developed by KMV)

- Is a structural model
- Adopts forward looking EDF and not historical ones
- Two companies with the same rating grade can have different default probabilities.
- Indeed a new rating grade is defined through the KMV approach
- For each new grade it follows the CreditMetrics approach

Other models

Credit Portfolio View (developed by McKinsey)

- Is a segment-structural model in the sense that it considers the company sector and the geographical area
- The probability of default is modeled through a Logit regression where the input are the sector and geographical indicators
- Thus it is a multivariate econometric model
- Default probabilities are linked with economic cycle
- The whole transaction matrix is linked with economic cycle as well

Other models

Credit Risk Plus (developed by Credit Swiss Financial Products)

- Is not a structural model
- It follows an actuarial point of view
- It considers only defaults, not downgrading
- It counts the number of expected defaults for each single rating grade
- Then the probability of default in each rating grade is modeled through a Poisson distribution.

Summary comparison

	CreditMetrics	Portfolio Manager	Credit Portfolio View	Credit Risk Plus
Type of risks	Migration, default, recovery	Migration, default, recovery	Migration, default, recovery	Default
Definition of risk	Variation in future market values	Loss from migration and default	Variation in future market values	Loss from default
Risk factors for transaction matrix	Rating grade	Distance to default point	Rating grade and economic cycle	(transaction not considered)
Transaction matrix	Historical and constant	Structural microeconomic model	Economic cycle	(transaction not considered)
Risk factors for correlation	Asset correlation based on equity correlation	Asset correlation based on equity correlation	Economic factors	Factor loadings
Sensitivity to economic cycle	Yes, through the downgrading	Yes, through the EDF estimated from equity values	Yes, through update of the transaction matrix	No, the default rate is volatile but not linked to economic cycle
Recovery rate	Fix or random (beta distribution)	Random (beta distribution)	Random (empirical distribution)	Deterministic
Adopted approach	Simulation	Simulation	Simulation	Analytic

Resti & Sironi (2005) - Rischio e valore nelle banche - Misura, regolamentazione, gestione
See also Resti & Sironi (2007) - Risk Management and Shareholders' Value in Banking: From Risk Measurement Models to Capital Allocation Policies